An Infrared Cutoff Revealed by the Two Years of *COBE*-DMR Observations of Cosmic Temperature Fluctuations

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Abstract

We show that a good fitting to the first two years of *COBE*-DMR observations of the two-point angular correlation function of CBR temperature is given by models with a non-zero infrared cutoff k_{min} in the spectrum of the primordial density perturbations. If this cutoff comes from the finiteness of the universe, say, a topological T3 model, we find $k_{min} \sim (0.3 - 1.1)\pi \mathrm{H}_0/c$ with confidence level 95%. Such a non-zero k_{min} universe would also give a better match to the observations both of the RMS quadrupole anisotropy of CBR and of galaxy clustering.

Almost half a century ago, Infeld and Schild [1] had pointed out that if the size of the universe is finite, the infrared cutoff k_{min} is then non-zero, and the infrared divergences in quantum field theory are automatically excluded. However, it was shown later that the problem of infrared divergence in QED can be solved even in an infinite universe ($k_{min} = 0$), because the infrared divergence in the radiative correction can be precisely eliminated by adding the soft photon bremsstrahlung to the vertex correction [2]. In this treatment we have, in fact, assumed $k_{min} < \Delta E/c\hbar$, where ΔE is the energy resolution of the detection system.

For experiments done in physics laboratory the condition of $k_{min} < \Delta E/c\hbar$ is always held. For instance, if the size of the universe is of the order of today's horizon $c\mathrm{H}_0^{-1} \sim 3000\ h^{-1}$ Mpc (where h is the Hubble constant in unit of 100 km $\mathrm{s}^{-1}\mathrm{Mpc}^{-1}$), in order to detect soft photons with energy $\sim k_{min}$ the energy resolution ΔE should be as small as $\hbar\mathrm{H}_0$. The uncertainty principle requires then that the time needed to measure the cosmic infrared cutoff is $\sim \mathrm{H}_0^{-1}$. Therefore, it is impossible to detect the cosmic infrared cutoff k_{min} by local experiments [3].

However, astrophysics do provide experiments, such as cosmic background radiation (CBR), which have lasted as long as the age of the universe, $\sim \mathrm{H}_0^{-1}$, and be able to measure k_{min} comparable with H_0/c. Using this idea, we proposed that the CBR anisotropy is an effective tool to detect the size of a small universe. We showed that the ratios of quadrupole moment of CBR anisotropy to higher order multipoles sensitively depend on the ratio $L/(c\mathrm{H}_0^{-1})$, where L is the size of the universe [4]. To compare this $L/(c\mathrm{H}_0^{-1})$-dependence with the first year data of the *COBE*-DMR [5], it has been found [6] that the lower limit to the size of a cubic T3 universe should be much larger than that given by the distributions of galaxies and QSOs [7].

In this letter, we will show that a non-zero k_{min} is possible. We are motivated by the Two Years of *COBE*-DMR observations of the CBR anisotropy (hereafter, the two-year data), which has been available recently [8]. Compared with the first year data [5], the quality of the two-year data on many aspects has been significantly improved. A new

result is found to be that the RMS quadrupole amplitude $Q_{rms} = 6 \pm 3\mu$K is significantly less than the most likely quadrupole-normalized amplitude Q_{rms-PS}, which is $12.4^{+5.2}_{-3.3}\mu$K for n=1.6, or $17.4 \pm 1.5\mu$K for $n = 1.0$, where n is the index of the power-law spectrum of density perturbation. This Q_{rms}-Q_{rms-PS} difference can not be totally explained by cosmic variance [8]. On the other hand, smaller Q_{rms} may indicate the lack of density fluctuations on the largest scales [4]. Therefore, it is valuable to study the possibility of explaining the Q_{rms}-Q_{rms-PS} difference by models with non-zero infrared cutoff k_{min}.

Since the two-year data were reduced in the scheme of standard inflationary universe ($\Omega = 1$, $\Lambda = 0$, and simply connected spatial hypersurface), we consider a Gaussian and adiabatic primordial perturbation with power-law spectrum in a flat universe. The large-scale fluctuations in the CBR temperature are given by [9]:

$$\frac{\Delta T}{T}(\mathbf{\Omega}) = -\frac{H_0^2}{2c^2}\sum_{\mathbf{k}} \frac{\delta(\mathbf{k})}{k^2} e^{-i\mathbf{k}\cdot\mathbf{y}}\,, \tag{1}$$

where $\mathbf{y} = (2cH_0^{-1}, \mathbf{\Omega})$ is a vector of length $2cH_0^{-1}$ pointing to the direction $\mathbf{\Omega}$ on the sky. $\delta(\mathbf{k})$ is the Fourier amplitude of the density contrast $\delta(\mathbf{r})$. For a power-law perturbation, $\langle|\delta(\mathbf{k})|^2\rangle$ can be written as [10]

$$\langle|\ \delta(\mathbf{k})\ |^2\rangle = \frac{(2\pi)^3}{V_\mu}\Delta_0^2\frac{1}{4\pi}k^n\,, \tag{2}$$

where V_μ is a large rectangular volume, $k = |\mathbf{k}|$ and Δ_0^2 is a constant determined by the variance of the perturbed potential.

The observed temperature fluctuations of CBR on the celestial sphere are usually expressed by spherical harmonics $\Delta T/T(\mathbf{\Omega}) = \sum_{lm} a_l^m Y_l^m(\mathbf{\Omega})$, where $Y_l^m(\mathbf{\Omega})$ are the spherical harmonic functions. Defining a rotationally invariant coefficient $C_l \equiv \sum_m \langle|a_l^m|^2\rangle$, one found from eqs.(1) and (2) that

$$C_l = \frac{H_0^4(2l+1)}{4c^4}\Delta_0^2\frac{2\pi^2}{V_\mu}\sum_{\mathbf{k}} k^{n-4} j_l^2(ky)\,, \tag{3}$$

where $j_l(x)$ is the spherical Bessel function. Since our defined C_l is dimensionless, the *COBE* quadrupole amplitude Q is related to our quadrupole amplitude C_2 by $Q = C_2^{1/2}T$, where T is the mean temperature of CBR.

If the universe is of $k_{min} = 0$, eq.(3) becomes [11]

$$C_l^0 = \frac{H_0^4(2l+1)}{4c^4}\Delta_0^2 \int_0^\infty k^{n-2} j_l^2(ky)dk\,, \tag{4}$$

where the lower limit of the integration in eq.(4) is taken to be zero, because $k_{min} = 0$. When $n < 3$, eq.(4) gives

$$C_l^0 = \frac{2l+1}{5} C_2^0 \frac{\Gamma(\frac{9-n}{2})\Gamma(l+\frac{n-1}{2})}{\Gamma(\frac{n+3}{2})\Gamma(l+\frac{5-n}{2})}\,, \tag{5}$$

where C_2^0 is the quadrupole moment in a $k_{min} = 0$ universe.

Let us consider a $k_{min} \neq 0$ universe, say, a cubic T3 universe [4], which is constructed from a flat and infinite universe by the following identification on the 3-dimensional flat hypersurface: $(x_1 + lL, x_2 + mL, x_3 + nL) = (x_1, x_2, x_3)$ for all integers l, m, n. L is the size of the universe. In this case, the coefficients C_l should be directly calculated from eq.(3), i.e.

$$C_l = \frac{H_0^4(2l+1)}{16\pi c^4}\Delta_0^2 y^{1-n}(yk_{min})^3 \sum_{\mathbf{k}} (ky)^{n-4} j_l^2(ky)\,, \tag{6}$$

where $k_{nim} = 2\pi/L$, and the summation covers all possible states of the wave vector: $\mathbf{k} = k_{min}(l, m, n)$. Obviously, C_l now depends on three parameters: 1) the amplitude Δ_0 of the perturbation (or the quadrupole moment $C_2^{1/2}$), 2) the infrared cutoff k_{min} (or the size of the universe L), and 3) the index n of the perturbation spectrum. When $k_{min} \to 0$, or $L \to \infty$, one has $C_l \to C_l^0$. Therefore, this k_{min}-dependence of C_l provides an effective method to detect non-zero infrared cutoff k_{min}.

In the *COBE*-DMR observations, two measurements are independent: a) the two-point angular correlation function $C(\theta)$ of CBR temperature, and b) the RMS quadrupole amplitude Q_{rms}. The two-point angular correlation function $C(\theta)$ is related to C_l by

$$C(\theta) = \sum_l C_l W^2(l) P_l(\cos\theta)\,, \tag{7}$$

where $P_l(x)$ is the Legendre function, and $W(l)$ is a window function, which is $W(l) = \exp\{-1/2[l(l+l)/17.8^2]\}$ [8]. Comparing the measured correlation function to the model of eqs.(5) and (7), one can find a most likely quadrupole-normalized amplitude Q_{rms-PS}.

The result based upon the first year data does show $Q_{rms-PS} \sim Q_{rms}$ [5]. Therefore, the model of $k_{min} = 0$, i.e. eqs.(4) and (5), is consistent with the first year data. However, for the two year data, the difference between the RMS quadrupole and the most likely normalized quadrupole is as large as $2-4\sigma$ (Even taking the cosmic variance into account, the difference is still significant at 90% confidence level for $n = 1$ [8]). Therefore, one would no longer be able to confidently say that the standard model ($n = 1$) is totally compatible with the current *COBE*-DMR observations.

We use the standard χ^2 technique to test the T3 models and to estimate the most likely model parameters (quadrupole amplitude C_2 and size L) by a χ^2-minimization over the two-point angular correlation function. For a given n and y/L, we estimate $C_{2\ rms-PS}$ by minimizing χ^2 over the data:

$$\chi^2 = \sum_i \frac{[C_i - C(\theta_i)]^2}{\sigma_i^2 + \sigma_{cv}^2(\theta_i)}, \tag{8}$$

where C_i and σ_i are, respectively, the observed values and errors of the angular correlation at θ_i, and $\sigma_{cv}(\theta)$ is the 1σ cosmic variance of the $C(\theta)$ [12]. In eq.(8) we assumed that the variances in different bins are mutually independent. This approximation is probably suitable for our purpose, because it has been known, at least for the first year date, that the best fit amplitude of quadrupole does not significantly depend on the nondiagonal part of the covariance matrix of C_i [13]. Since σ_{cv} is also proportional to $C_{2\ rms-PS}$, we adopt an iteration procedure to calculate $C_{2\ rms-PS}$. First we assume a zero σ_{cv} and find out the best-fitting value of $C_{2\ rms-PS}$. Using this value we calculate the σ_{cv} based on 100 Monte Carlo realizations of C_l, and do the minimization again and find out a new fitting value of $C_{2\ rms-PS}$. Using this new $C_{2\ rms-PS}$ we repeat the minimization and find another more accurate value of $C_{2\ rms-PS}$. The iteration procedure is stopped until the differences of χ^2 and of $C_{2\ rms-PS}$ between the two consecutive minimizations are less than 0.1%. The final $C_{2\ rms-PS}$ and χ^2 are our desired values. In fact the calculation converges very fast, and we need less than 5 iterations. Fig.1 shows the goodness of the χ^2-fit of each model, i.e. the probability $P(> \chi^2_{min})$ that the experimental data are drawn from a realization of the model. It can be seen from Fig.1 that for $k_{min} \sim 0$

models, i.e. $L \gg y = 2cH_0^{-1}$, the $P(>\chi^2_{min})$ of $n = 1.6$ is much greater than that of $n = 0.6$ or 1.0. This result is the same as that of likelihood analysis done by the *COBE* group. Although the $n = 1.6$ case can be comfortably accommodated by the two-year data [acceptance probability $P(>\chi^2_{min}) \sim 40\%$], the smaller indices $n \le 1$, which are favored more by current observational data of galaxy clustering [14], have much lower acceptance probability.

Differing from the first year data, a remarkable feature shown in Fig.1 is the high peak in $P(>\chi^2_{min})$ space at $y/L \sim 0.8$. This means that the best fit value of the infrared cutoff is $k_{min} \sim 0.8\pi H_0/c$. In Fig.2 we plot such a best-fitting curve as well as the two-year data of $C(\theta)$. We also plot the best-fitting curve of an $k_{min} = 0$ universe ($n = 1.6$) in Fig.2. It can be clearly seen from both Figs.1 and 2 that considering the possible existence of a non-zero cosmic infrared-cutoff substantially improves the model fit to the experimental data (even for $n = 1.6$), and the most probable value of y/L is $\sim$0.8, which is almost independent of n. At the 95% confidence level, we have $0.3 < y/L < 1.1$ for $n = 1$, $0.5 < y/L < 1.1$ for $n = 0.6$ and $y/L < 1$ for $n = 1.6$.

Fig.3 plots our best-fitting $C^{1/2}_{2\ rms-PS}$ as a function of y/L, where the index n is taken to be 0.6, 1.0 and 1.6 as well. We plot the measured value of $C^{1/2}_{2\ rms}$ by the bold line in Fig.3, and its 1σ range by the dotted area. One can find that the models with $L < 1.2y = 2.4cH_0^{-1}$ give good agreement between Q_{rms-PS} and Q_{rms}, i.e. their difference is no longer larger than 1 σ. This result is also almost independent of the power law index n.

Because a finite universe lacks the fluctuation power on scales larger than its size, we require a larger amplitude Δ_0 of the density perturbation to fit with the observational $C(\theta)$ [15]. However, larger Δ_0 will lead to a stronger clustering on smaller scales. Therefore we should study whether the amplitude Δ_0^2 in a $k_{min} \ne 0$ universe is compatible with observed clustering of galaxies on scales, say, $8\,h^{-1}$Mpc. In order to calculate the density fluctuations on smaller scales from a given Δ_0 perturbation, we need to choose the transfer function $T(k)$ of linear growth [9]. The $T(k)$ in turn is mainly determined by the matter

composition of the universe. Here we use the $T(k)$ of the standard cold dark matter (CDM) model and of the cold plus hot dark matter (CHDM) model ($\Omega_{CDM} = 0.7$, $\Omega_\nu = 0.3$, and $h = 0.5$ [16]). Fig.4 presents the predicted values of σ_8, the rms density fluctuation of a sphere $r = 8\,h^{-1}$Mpc, as a function of y/L for $n = 1$. In Fig.4 we also plot a result of σ_8 given by statistics of galaxy distribution. By examining clusters of galaxies, White et al. [17] suggest that in an Einstein-de Sitter universe σ_8 is between 0.52 and 0.62. These limits are shown as the solid lines in Fig.4. Obviously, small universes of $y/L \gg 1$ are not acceptable, because they produce excessive clustering on $8\,h^{-1}$Mpc scale for both $T(k)$. The models of $y/L \leq 1$ are acceptable when the CHDM $T(k)$ is used. [Considering uncertainties in the estimated limits of σ_8 and in the choice of $T(k)$, we caution readers against placing overaccurate constraints of the σ_8-test on the universe sizes].

With reasonable transfer functions, the best-fitting value of power index $n = 1.6$ for large universe ($L \gg 2cH_0^{-1}$) is hard to reconcile with the observational data of galaxy clustering [14]. While the *COBE*-DMR data allows the finite universe of $L \sim 1.2cH_0^{-1}$ to have $n = 1$, which can be brought into agreement with galaxy clustering for some reasonable transfer functions. Therefore, compared with models of $k_{min} = 0$ (infinite) and large k_{min} (small) universes, the moderate-size (or $k_{min} \sim \pi \mathrm{H}_0/c$) universe is in better agreement with the observations when both the *COBE*-DMR result and the clustering of galaxies are considered.

In summary, in terms of the three independent and basic tests, i.e. $C(\theta)$, Q_{rms} and the galaxy clustering, a good survived model among those discussed in the paper is the one of a non-zero k_{min} universe. If this cutoff comes from the finiteness of the universe, such as a topological T3 model, the most probable value of k_{min} is $0.8\pi H_0/c$. However, it does not mean that the $k_{min} \neq 0$ can only be given by a multiply connected topology like T3. There are other mechanism for $k_{min} \neq 0$. For instance, in the standard inflationary universe, if the parameter N_{tot} in the inflation factor $\exp(N_{tot})$ is ~ 54 (somewhat of fine-tuning parameter may be needed), one would also have an initial density perturbation with spectrum cutted off at $k_{min} \sim \pi H_0/c$. Regardless of these variousness of possible

explanations, one can conclude, at least, that the *COBE*-DMR observation of $\Delta T/T$ is a powerful tool to probe the cosmic infrared cutoff. The first two years of the *COBE*-DMR observations reveal the possible existence of non-zero infrared cutoff in the spectrum of the primordial density fluctuations. It seems to be the first time to derive an "observed" values of the cosmic k_{min} with confidence higher than 95%. However, we should remember the difficulty in the measurement of Q_{rms} due to foreground contaminations. Therefore, in order to discriminate various explanations, it is necessary to have other tests. The amplitude of octavopole C_3 of $\Delta T/T$ would be one of such tests [18]. We believe, more convinced results related to the non-zero cosmic infrared cutoff will be obtained as more observations of CBR $\Delta T/T$ become available.

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Figure captions

Fig.1 $P(>\chi^2_{min})$ as a function of y/L. Here χ^2_{min} is calculated by fitting the finite L universe model to the *COBE*-DMR two-point angular correlation function of the cosmic temperature fluctuations. The index n of power law perturbations is taken to be 0.6, 1.0 and 1.6. The dot-dash line denotes $P(>\chi^2_{min}) = 5\%$.

Fig.2 The observed angular correlation $C_l(\theta)T^2$ and the best-fit curves by a) a T3 model with $n = 1$ and $y/L = 0.80$ (solid line), and b) an infinite and flat universe with $n = 1.6$ (dashed line).

Fig.3 The best fitted quadrupole, i.e. $C^{1/2}_{2\ rms-PS}$, as a function of the size of the universe. The thick line denotes the RMS quadrupole measurement $C^{1/2}_{2\ rms}$, and the dotted area is its 1σ region.

Fig.4 The predicted σ_8 in a cubic T3 models. Two types of transfer functions are assumed: the standard CDM (dotted line) and the CHDM (dashed line). The solid lines are the upper and lower limits to σ_8, given by statistics of galaxy distribution [17].